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Modeling Evacuees' Exit Selection with Best Response Dynamics

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Summary. We present a model for occupants' exit selection in emergency evacuations. The model is based on the game theoretic concept of best response dynamics, where each player updates his strategy periodically according to other players' strategies. A fixed point of the system of all players' best response functions defines a Nash equilibrium of the game. In the model the players are the occupants and the strategies are the possible target exits. We present a mathematical formulation for the model and analyze its properties with simple test simulations.

1 Introduction

Selection of the exit route is one of the most important decisions that occupants will face in an emergency evacuation. This decision is influenced by many factors, such as personal characteristics, building geometry, and observations concerning the dynamic evacuation situation. It is natural that one of the evacuees' main goals is to get out of the building as fast as possible. On the other hand, evacuees tend to prefer familiar alternatives, because they feel that unknown alternatives increase the threat [1]. The visibility of exits also influences the decisions of occupants, since the information of population and conditions at an exit are limited if the exit is not visible [2]. In fire evacuations, occupants will naturally avoid the routes that are smoky or in flames. After all, the ultimate goal of evacuees is to get out of the building alive and the above mentioned tendencies are just means of achieving this goal.

The decisions that occupants make on their exit routes will affect the outcome of the evacuation. Therefore, it is important to take these decisions into account in evacuation simulation models. Some evacuation models make the assumption that all evacuees will head straight to the nearest exit at the start of evacuation. Also many prescriptive fire codes implicitly assume that the total exit width of buildings is used in evacuation. Experience and studies have shown that this assumption is unrealistic in many occasions [1, 3]. Let us briefly recall some previous studies in the field of exit selection modeling. The exit selection model of buildingEXODUS [2] uses an adaptive decision making model, where the evacuees are allowed to change their target exit a few times during the evacuation. This model considers several factors that influence the decision, e.g., occupants' familiarity with the exits, visibility of exits and the lengths of queues at the exits. In a subsequent article also the effect of fire conditions on exit selection have been considered [4]. The buildingEXODUS model uses a heuristic approach, and specific formulas and parameters of the model have not been published. Lo *et.* al [5] presented a game theoretic approach for exit selection. The model simplifies the exit selection situation to a two player zero sum game, where the crowd is considered to play a "virtual entity".

In this article, we present a game theoretic model for evacuees' exit selection. The model is based on the game theoretic concept of players' best response functions. A fixed point of the system of these functions defines a *Nash equilibrium* of the game. In our game theoretic model, we interpret evacuees' updating of their best response actions as an adaptive dynamical model. The model has two stages: on one hand the evacuees try to select the fastest exit route. On the other hand, there are also other factors affecting the decision making, like smokiness, familiarity, and visibility of exits. These factors are taken into account by adding constraints to the evacuation time minimization problem.

It is quite obvious that factors like distances to the exits, queue length, and the visibility of the exits need to be taken into account in an exit selection model. These things are considered in our model as well as in some previous approaches. The key contribution of this article is to formulate and analyze the model in a game theoretic framework. We present a mathematical formulation for the reaction function model. We also analyze the emerging phenomena, such as convergence to a Nash equilibrium, using numerical simulations.

2 The Model and a Game Theoretic Formulation

In this chapter we present a game theoretic model for exit selection. In the model, the occupants update their decisions based on their best response functions. A fixed point of the system of these functions is a Nash equilibrium of the game. Best response dynamics have been successfully used in many fields of science, e.g., in telecommunications networks [6, 7] and various road traffic situations [8].

To formulate our exit selection model as an *N-player game* in a normal form, we begin by defining the concepts of best response and Nash equilibrium. For thorough explanations of the concepts, see [9].

2.1 An N-Player Game

In a *normal form static game*, each of the N players, or *agents*, playing the game selects a strategy s_i, where i refers to agent i. Let S_i be the set of all strategies for agent i, so that $s_i \in S_i$. The payoff of the game for agent i is a function of the strategies of all players. This function is called *payoff function* and it is denoted by $u_i(s_1, \dots, s_n)$. The objective of each player is to select the strategy, which maximizes his own payoff, given that also other players maximize their payoffs. In an implementation of this one-stage game the players act according to their maximizing strategies.

A *Nash equilibrium* (NE) of the game is a profile of strategies $(s_1^*, \dots, s_n^*)$ such that each player's strategy is an optimal response to the other players' optimal strategies. Hence, a strategy profile $s^* = (s_1^*, \dots, s_n^*)$ is a NE if s_i^* solves

$$s_i^* = \arg \max_{s_i \in S_i} \quad u_i(s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots, s_n^*) \tag{1}$$

for all i. This means that no player can profit by deviating from NE if the others play the NE strategies. When the sets S_i are finite, a game may not have a Nash equilibrium in *pure strategies*, but in *mixed strategies*, i.e., when the strategies are distributions over the sets of pure strategies S_i, any game has at least one equilibrium. This result was shown by John Nash in his seminal paper in 1950 [10].

The *best response function* of player i is defined by

$$s_i := BR_i(s_{-i}) := \arg \max_{s_i' \in S_i} u_i(s_i', s_{-i}), \tag{2}$$

where $s_{-i} := (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$. This function defines the strategy s_i that is the *best response* of player i to the the other players' strategies, s_{-i}, i.e., the strategy that maximizes the payoff of player i when the others play s_{-i}.

The best response function is also called *best response correspondence*, since $BR(s_{-i})$ can be a set. It is easy to show that if a strategy profile $\bar{s} = (\bar{s}_1, \dots, \bar{s}_n)$ satisfies the equation

$$\bar{s}_i = BR_i(\bar{s}_{-i}), \text{ for all } i, \tag{3}$$

then $\bar{s}$ is a NE of the game. Mathematically, note that $\bar{s}$ is a *fixed point* of the system of all players' best response correspondences.

Under suitable assumptions, an iterative process, where players update their strategies according to their best response correspondences, will converge to a Nash equilibrium [11]. In this paper we shall consider a familiar fixed point iteration to define the NE and interpret it as an adaptive process for the exit selection dynamics.

2.2 Exit Selection Model

In the exit selection model we assume that the occupants tend to select the exit route through which the evacuation is the fastest. However, there are also other factors influencing the decision. We will include three other factors in this model: familiarity and visibility of the exits and the fire related conditions at the exits.

To calculate an estimate for an agent's evacuation time through an exit, one needs to consider two things, the distance to the exit and the queue length in front of the exit. Thus, the *estimated evacuation time* of an agent is calculated as the sum of *estimated moving time* and *estimated queuing time.* The moving time is estimated simply by dividing the distance to the exit by the walking speed of the agent.

The queuing time of an agent at an exit depends on the width of the exit and on the number of the other agents that are heading to that exit and are closer to it than the agent itself. Adding the queue length into the model in a fashion where the queuing time of an agent depends not only on the locations of other agents but also on their target exits, makes the decision of an agent dependent on the decisions of the others. This makes this model a game model.

During an evacuation, the fastest exit may change. In these situations the agents should be able to react to the new situation and change their target exits. This is modeled by updating the best response functions of each agent in certain periods of time.

The familiarity, visibility, and conditions at the exit are taken into account by constraining the set of feasible exits according to these factors. These factors divide the exits into six groups that have a preference order. Each agent will select an exit from the nonempty group that has the highest preference. If there are several exits in this group, the selection is made by minimizing the evacuation time as presented above.

2.3 Mathematical Formulation of the Model

We refer to the agents with indices i and j, where $i, j \in \mathcal{N} = \{1, 2, \ldots, N\}$. The strategies of the agents are the exits e_k, $k \in \mathcal{K} = \{1, 2, \ldots, K\}$. We shall also use the notation $s_i \in \{e_1, \ldots, e_K\} = S_i$, $i \in \mathcal{N}$ for strategies and strategy sets. We denote the profile of all agents' strategies by

$$s := (s_1, \ldots, s_N) \in S_1 \times \cdots \times S_N = S, \tag{4}$$

and will also use notation $s_{-i} := (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_N) \in S_{-i}$ for the strategies of all other agents but agent i, and notation (s_i, s_{-i}) for the whole strategy sequence s.

Let us denote the positions of agent i and exit e_k by $\mathbf{r}_i$ and $\mathbf{b}_k$, respectively, and let $\mathbf{r} := (\mathbf{r}_1, \ldots, \mathbf{r}_N$. Agent i's distance from exit e_k is

$$d(e_k; \mathbf{r}_i) = ||\mathbf{r}_i - \mathbf{b}_k||. \tag{5}$$

Now, the payoff function of agent i is the estimated time of evacuation, $T_i(s_i, s_{-i}; \mathbf{r})$, which he attempts to minimize. It is the sum of estimated queuing time and estimated moving time. When agent i chooses strategy $s_i = e_k$, T_i is evaluated as

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$$T_i(e_k, s_{-i}; \mathbf{r}) = \beta_k \lambda_i(e_k, s_{-i}; \mathbf{r}) + \tau_i(e_k; \mathbf{r}_i), \tag{6}$$

where β_k is a scalar describing the capacity of exit e_k, $\lambda_i(e_k, s_{-i}; \mathbf{r})$ is the number of other agents that are heading to the same exit e_k as agent i and are closer to it, and $\tau_i(e_k; \mathbf{r}_i)$ is the estimated moving time of agent i to exit e_k. The function λ_i is defined by $\lambda_i(e_k, s_{-i}; \mathbf{r}) = |\Lambda_i(e_k, s_{-i}; \mathbf{r})|$, where

$$\Lambda_i(e_k, s_{-i}; \mathbf{r}) = \{j \neq i | s_j = e_k\,,\, d(e_k; \mathbf{r}_j) \leq d(e_k; \mathbf{r}_i)\}, \tag{7}$$

and $|\cdot|$ denotes the number of elements in a subset of $\mathbb{N}$.

The estimated moving time to an exit is calculated by

$$\tau_i(e_k; \mathbf{r}_i) = \frac{1}{v_i^0} d(e_k; \mathbf{r}_i), \tag{8}$$

where v_i^0 is the moving speed of agent i. The strategy of agent i is the best response to the other agents' strategies:

$$s_i = BR_i(s_{-i}; \mathbf{r}) = \arg \min_{s_i' \in S_i} T_i(s_i', s_{-i}; \mathbf{r}). \tag{9}$$

A Nash equilibrium of the game satisfies $s_i^* = BR_i(s_{-i}^*; \mathbf{r})$ for all i.

The effects of *familiarity, visibility* and *fire related conditions* are taken into account by defining three binary variables

$$fam_i(e_k),\ vis(e_k; \mathbf{r}_i),\ con(e_k; \mathbf{r}_i), \quad \forall i \in \mathcal{N},\ k \in K,$$

where

$$fam_i(e_k) = \begin{cases} 1, \text{ if exit } e_k \text{ is familiar to agent } i \\ 0, \text{ if exit } e_k \text{ is not familiar to agent } i \end{cases}$$

$$vis(e_k; \mathbf{r}_i) = \begin{cases} 1, \text{ if exit } e_k \text{ is visible to agent } i \\ 0, \text{ if exit } e_k \text{ is not visible to agent } i \end{cases}$$

$$con(e_k; \mathbf{r}_i) = \begin{cases} 1, & \text{conditions are tolerable at exit } e_k \text{ for agent } i \\ 0, & \text{conditions are intolerable at exit } e_k \text{ for agent } i \end{cases}$$

Now the exits can be divided into groups that have preference numbers from one to six according to the values of these binary variables. The smaller the preference number is, the more preferable the exit. Definitions for these numbers are presented in Table 1. The familiarity of an exit is considered to be more important to the agents than the visibility. This is based on social psychological findings, according to which evacuees prefer familiar routes even if there were faster unfamiliar routes available [1, 3].

Hence, the complete exit selection model can be presented for each agent $i \in \mathcal{N}$ as follows:

$$s_i = BR_i(s_{-i}; \mathbf{r}) = \arg \min_{s_i' \in S_i} T_i(s_i', s_{-i}; \mathbf{r}), \qquad \text{st. } s_i' \in E_i(\bar{z}), \tag{10}$$

where $E_i(\bar{z})$ is the non-empty exit group with the best preference number $\bar{z}$ for agent i.

preference number	exit group	$vis(e_k, \mathbf{r}_i)$	$fam_i(e_k)$	$con(e_k, \mathbf{r}_i)$
1	$E_i(1)$	1	1	1
2	$E_i(2)$	0	1	1
3	$E_i(3)$	1	0	1
4	$E_i(4)$	1	1	0
5	$E_i(5)$	0	1	0
6	$E_i(6)$	1	0	0
	No preference	0	0	1
	No preference	0	0	0

Table 1. The preference numbers of exit groups used in our model. The smaller the preference number is, the more preferable the exit. The combinations of the last two rows have no preference. This is because the evacuees are unaware of the exits that are unfamiliar and invisible, and thus cannot choose these exits.

2.4 Additional Features of the Model

There are some other matters that need to be taken into account in the exit selection model but are not included in the basic formulation above.

Sometimes an alternative exit is only slightly faster than the current target exit. We assume that an agent may not always notice the small difference, or may not react to them. This is why a *patience parameter* is added to the model. The parameter describes how much faster an alternative exit needs to be in order for an agent to change its target exit. This behavior can be taken into account by subtracting the patience parameter from the evacuation time through the current target exit. Another possibility would be to define the patience parameter as a proportion of the estimated evacuation time, instead of absolute seconds. In this case the estimated evacuation time of the current exit is multiplied by the parameter, which can have values between zero and one.

In some situations, an agent may not be able to estimate the queue length in front of an exit. This is especially the case in situations where the agent cannot see the exit. In these cases the estimated evacuation time should not depend on the queuing time, and thus, Eq. (6) should be replaced by

$$T_i(e_k, s_{-i}; \mathbf{r}) = \beta_k \lambda_i(e_k, s_{-i}; \mathbf{r}) + vis(e_k, \mathbf{r}_i)\tau_i(e_k; \mathbf{r}_i). \tag{11}$$

This makes the estimated evacuation times shorter for the invisible exits. However, this does not affect the functioning of the model, because the estimated evacuation times are only compared between exits in the same exit group.

3 Computational Results

The presented exit selection model has been implemented to the FDS+Evac software [12, 13], which enables the use of fire related data in the model. However, in this article we study its computational properties using a simple cellular automata based program. The focus of our analysis is on the convergence properties of the iterative model. We shall use fixed point iteration to simulate the convergence of best response dynamics to a Nash equilibrium. The iterative process can be presented with an equation as follows:

$$s_i^{t+1} = BR_i(s_{-i}^t; \mathbf{r}),\ \forall i \in \mathcal{N},\ t \geq 1. \tag{12}$$

Thus, in every iteration round the strategy of agent i is his best response to the other players strategies in the previous round. Basically, this means that all agents are considered to update their strategies simultaneously. This *parallel update algorithm* is not the only possible method for updating a best response algorithm. For descriptions of other possible approaches, see [6].

In the following examples, we present numerical results on the convergence of the model in a situation, where the agents do not move during the iteration. The agents just update their target exits at each iteration as best responses to the other agents' decisions. Basically, these iterations could be interpreted as situations where, at each iteration round, all agents tell each other the exit they are heading to. Then, in the next round, they all update their target exits as best responses to the strategy profile of the previous round. It turns out that in fairly large agent populations, the iteration converges to a NE with a quite small number of iterations. The fast convergence is a little bit astonishing, since fixed point iterations do not always converge very well for games with pure strategies [9].

Figure 1 shows how the iteration converges to a NE in a simple test geometry. Total of 100 agents are located randomly into a square $40\,\mathrm{m} \times 40\,\mathrm{m}$ room with two exits. Both of the exits are on the same wall and they are represented in the figures with large circles. The circles representing the left-hand and right-hand exits are white and black, respectively. The white exit is twice as wide as the black exit. The smaller white and black circles represent the agents and their target exits. In the initial position, the target exit of each agent is picked randomly. At each iteration, the agents update their target exits as a best responses to the current situation. In this example, the iteration converged into a NE in five iterations. In the equilibrium, a small majority of the agents are heading to the wider white exit. The widths of the exits are not very significant in this simulation, because the population density is quite small, and thus, the estimated queuing times are small relative to the estimated moving times. As population density increases, the queuing times increase relative to the moving times and the exit width becomes more important. As a result, a larger proportion will select the wider exit as population density increases. This can be seen in Fig. 2, which shows Nash equilibria for the same geometry with 300 and 500 agents.

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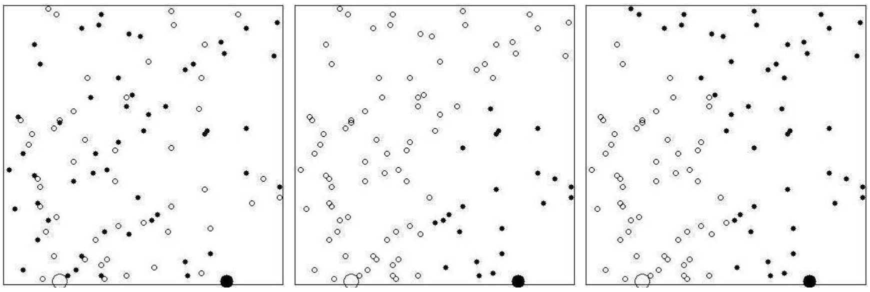
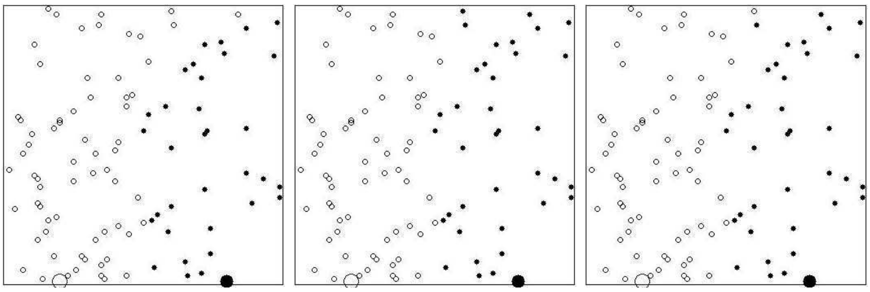
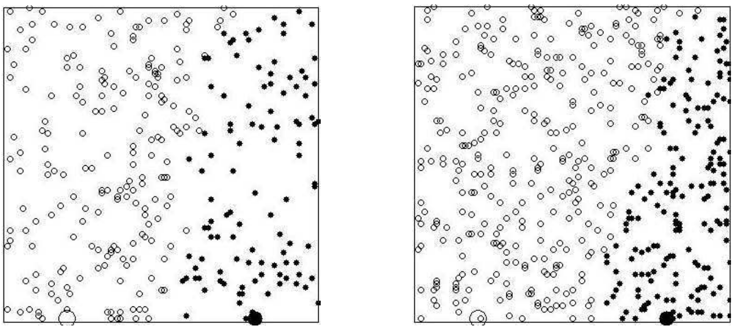


(a) Initial situation (b) After 1st iteration (c) After 2nd iteration



(d) After 3rd iteration (e) After 4th iteration (f) After 5th iteration. Equilibrium.

Fig. 1. An example of the convergence of the iteration.



(a) NE for 300 agents. (b) NE for 500 agents.

Fig. 2. Nash equilibria for simulations with 300 and 500 agents.

The graph in Fig. 3 describes the dependence between the number of agents and the average number of iterations needed to achieve the equilibrium. The number of iterations seems to increase linearly and even in large crowds the amount is very reasonable.

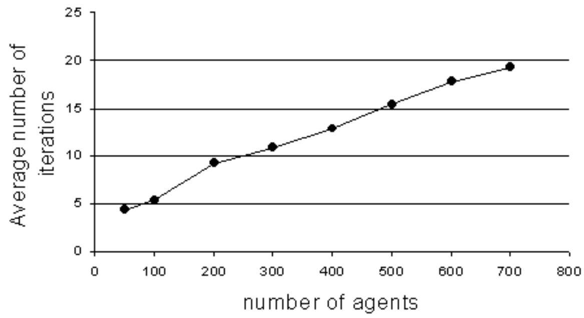


Fig. 3. The average number of iterations to NE versus the number of the agents. In these simulations the patience parameter is set to zero.

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It is quite obvious that the value of the patience parameter affects the number of iterations needed to achieve a NE. This is because as the parameter value increases, some strategy profiles that were not equilibria with a smaller patience parameter become equilibria. Figure 3 shows the dependence between the patience parameter and required iterations in the simple test geometry. The number of iterations rapidly decreases as the value of the parameter is increased from zero. As a consequence the equilibrium becomes really fast to compute when the parameter has nonzero values.

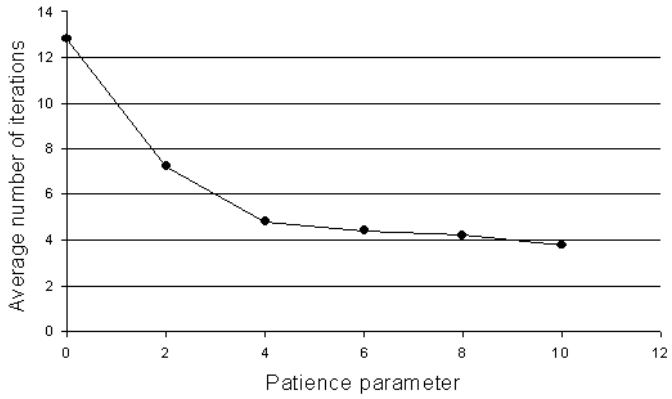

Fig. 4. The average number of iterations to NE for 400 agents versus the patience parameter. A unit measure of the patience parameter is one second.

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4 Discussion

We introduced a game theoretic reaction function model for evacuees' exit selection. The model considers several factors influencing the decision maker, such as distances to the exits, amount of crowd in front of the exits, and familiarity and visibility of the exits. The effects of these factors have been discussed in some previous articles [2, 4], however our approach is somewhat different as we formulate and analyze the model in a game theoretic framework. In the model, we interpret the exit selection of agents as an adaptive dynamical process, where agents update their decisions according to their best response functions and take an action accordingly. A Nash equilibrium of the game is a fixed point of the system of all agents' best response functions.

We also presented simple test simulations to analyze the properties of the model. It was found out that an iterative implementation of the model produces a Nash equilibrium with a reasonable number of iterations. When the patience parameter was added to the model the number of required iterations reduced rapidly as the parameter value was increased from zero. The effect of the parameter is natural, because as the value of the parameter is increased, some strategy profiles that were not equilibria with smaller parameter values become equilibria. Fixed point iterations do not always converge for games with pure strategies, and thus, analysis of the convergence properties of our model is a interesting topic for future research.

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